Extending Explicit Guidance Methods to Higher Dimensions, Additional Conditions, and Higher Order Integration

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Outline

1. Introduction
2. Original Formulation for E Guidance
3. Description of E Guidance Extensions
4. Simulation Results
5. Conclusion

Introduction

https://www.nasa.gov/sites/default/files/thumbnails/image/apollo_50th_full_color_72dpi.png

https://www.nasa.gov/sites/default/files/thumbnails/image/virtual_backgrounds_-_astronaut_step_1.jpg

Introduction

1. Explicit (E) Guidance: fundamentals for Apollo lunar guidance
2. Goal for E Guidance [1]
   a. Solve rendezvous two-point boundary value problem for rocket-propelled spacecraft at a specified terminal time
   b. Initial conditions: current position and velocity
   c. Terminal conditions: desired position and velocity
3. Original formulation for rockets but applicable to all vehicles
4. **Describe proposed E Guidance extensions in Ref. [2]**

[1] Cherry, G.: A general, explicit, optimizing guidance law for rocket-propelled spaceflight. In: Astrodynamics Guidance and Control Conference, p. 638 (1964)

[2] Kawamura, E.: Integrated targeting, guidance, navigation, and control for unmanned aerial vehicles. Ph.D. dissertation, University of Hawai'i at Manoa (2020).

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Original Formulation for E Guidance [1]

Goal: provide thrust (control function) from initial (0) states to desired (D) states

Commanded thrust acceleration:
$$\boldsymbol{a}_T(t) = [a_{Tx}, a_{Ty}, a_{Tz}]$$

Initial (0) states:
$$\boldsymbol{p}(t_0) = [p_{x,0}, p_{y,0}, p_{z,0}]$$
$$\mathbf{v}(t_0) = [v_{x,0}, v_{y,0}, v_{z,0}]$$

Desired (D) states:
$$\boldsymbol{p}(T) = [p_{x,D}, p_{y,D}, p_{z,D}]$$
$$\mathbf{v}(T) = [v_{x,D}, v_{y,D}, v_{z,D}]$$

[1] Cherry, G.: A general, explicit, optimizing guidance law for rocket-propelled spaceflight. In: Astrodynamics Guidance and Control Conference, p. 638 (1964)

Original Formulation for E Guidance [1]

1. Translational commanded thrust acceleration, $a_{Tx}(t)$, in the x-direction

$$a_{Tx}(t) = c_1 p_1(t) + c_2 p_2(t) - g_x(t) \quad (1)$$

   a. $p_1(t), p_2(t)$: linearly independent functions
   b. $c_1, c_2 \in \mathbb{R}$
   c. $g_x(t)$: x-component of acceleration due to gravity

2. The linearly independent functions can be polynomials

$$p_1(t) = \tau^m, p_2(t) = \tau^n \quad (2)$$

   a. $m, n \in \mathbb{Z}$ (integers)
   b. $\tau = (T - t)$
   c. T: terminal time
   d. t: current time
   e. T_{go}: time-to-go to reach the desired position and velocity

[1] Cherry, G.: A general, explicit, optimizing guidance law for rocket-propelled spaceflight. In: Astrodynamics Guidance and Control Conference, p. 638 (1964)

Original Formulation for E Guidance [1]

Set boundary conditions → Choose $p_1(t), p_2(t), T_{go}$ → Compute $\boldsymbol{F}$ matrix by integrating $p_1(t), p_2(t)$

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e F matrix

Initial (0) states:

$$\boldsymbol{p}(t_0) = [p_{x,0}, p_{y,0}, p_{z,0}]$$
$$\mathbf{v}(t_0) = [v_{x,0}, v_{y,0}, v_{z,0}]$$

Desired (D) states:

$$\boldsymbol{p}(T) = [p_{x,D}, p_{y,D}, p_{z,D}]$$
$$\mathbf{v}(T) = [v_{x,D}, v_{y,D}, v_{z,D}]$$

→ Integrate accelerations to get velocities → Integrate velocities to get positions

[1] Cherry, G.: A general, explicit, optimizing guidance law for rocket-propelled spaceflight. In: Astrodynamics Guidance and Control Conference, p. 638 (1964)

Original Formulation for E Guidance [1]

Set boundary conditions

Choose $p_1(t), p_2(t), T_{go}$

Compute $\boldsymbol{F}$ matrix by integrating $p_1(t), p_2(t)$

Con

ompute atrix and

e $\boldsymbol{E}$ matrix by the F matrix

Functions:

$p_1(t) = T - t$

$p_2(t) = (T - t)^2$

Terminal time: T = 10 s

$T_{go} = T - t$

Integrate accelerations to get velocities

Integrate velocities to get positions

[1] Cherry, G.: A general, explicit, optimizing guidance law for rocket-propelled spaceflight. In: Astrodynamics Guidance and Control Conference, p. 638 (1964)

Original Formulation for E Guidance [1]

Set bound
conditio

Compute accel
commands
acceleratic

Compute $\boldsymbol{F}$ matrix by integrating $p_1(t), p_2(t)$

Compute $\boldsymbol{E}$ matrix by inverting the F matrix

ate velocities
get positions

$$f_{11} = \int_{t_0}^{T} p_1(t)dt\,,$$

$$f_{12} = \int_{t_0}^{T} p_2(t)dt\,,$$

$$f_{21} = \int_{t_0}^{T} \left[\int_{t_0}^{t} p_1(s)ds\right] dt\,,$$

$$f_{22} = \int_{t_0}^{T} \left[\int_{t_0}^{t} p_2(s)ds\right] dt\,,$$

$$\Delta = \det(\boldsymbol{F}) = f_{11}f_{22} - f_{12}f_{21} \quad (3)$$

[1] Cherry, G.: A general, explicit, optimizing guidance law for rocket-propelled spaceflight. In: Astrodynamics Guidance and Control Conference, p. 638 (1964)

Original Formulation for E Guidance [1]

Set boundary

Choose

Compute $\boldsymbol{F}$ matrix by integrating $p_1(t), p_2(t)$

$$e_{11} = \frac{f_{22}}{\Delta}, e_{12} = -\frac{f_{12}}{\Delta}, \mathrm{e}_{21} = -\frac{\mathrm{f}_{21}}{\Delta}, \mathrm{e}_{22} = \frac{\mathrm{f}_{11}}{\Delta} \quad (3)$$

Compute $\boldsymbol{E}$ matrix by inverting the F matrix

Compute coefficients using $\boldsymbol{E}$ matrix and boundary conditions

Compute acceleration commands and accelerations

Integrate accelerations to get velocities

Integrate velocities to get positions

[1] Cherry, G.: A general, explicit, optimizing guidance law for rocket-propelled spaceflight. In: Astrodynamics Guidance and Control Conference, p. 638 (1964)

Original Formulation for E Guidance [1]

Set boundary condition

Choose

Compute $\boldsymbol{F}$ matrix by tegrating $p_1(t), p_2(t)$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \begin{bmatrix} \dot{x}_D - \dot{x}_0 \\ x_D - x_0 - \dot{x}_0 T_{go} \end{bmatrix} \quad (4)$$

Compute acceleration commands and accelerations

Compute coefficients using $\boldsymbol{E}$ matrix and boundary conditions

Compute $\boldsymbol{E}$ matrix by inverting the F matrix

Integrate accelerations to get velocities

Integrate velocities to get positions

[1] Cherry, G.: A general, explicit, optimizing guidance law for rocket-propelled spaceflight. In: Astrodynamics Guidance and Control Conference, p. 638 (1964)

Original Formulation for E Guidance [1]

Set boundary conditions

Choose

Compute $\boldsymbol{F}$ matrix by integrating $p_1(t), p_2(t)$

$$a_{Tx}(t) = c_1 p_1(t) + c_2 p_2(t) - g_x(t)$$

Compute $\boldsymbol{E}$ matrix by inverting the F matrix

Compute coefficients using $\boldsymbol{E}$ matrix and boundary conditions

Compute acceleration commands and accelerations

Integrate accelerations to get velocities

Integrate velocities to get positions

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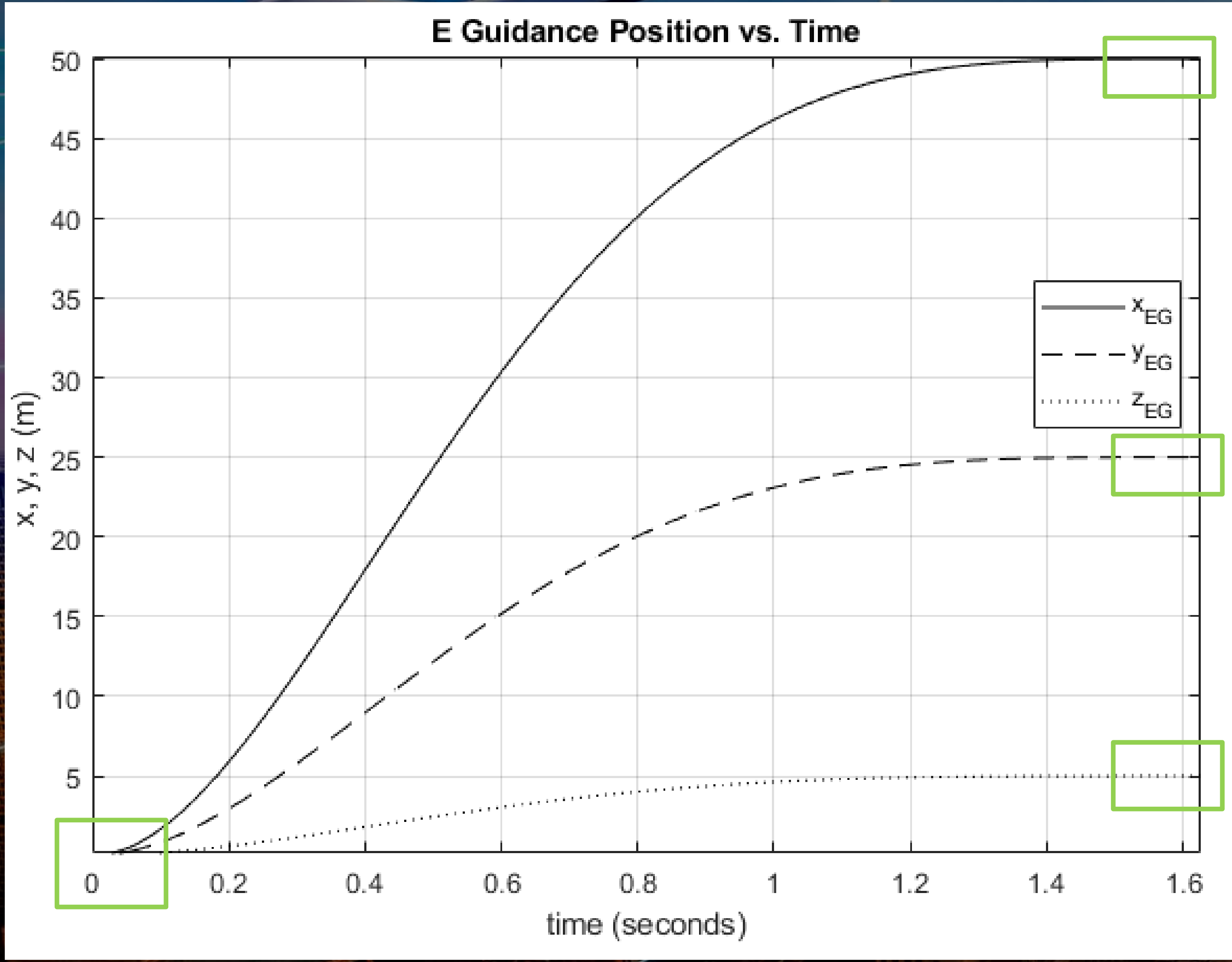
Original Formulation for E Guidance: Example



1. Initial position: [0,0,0]
2. Desired position: [50,25,5]
3. Selections
   a. $p_1(t) = (T - t)^2$
   b. $p_2(t) = (T - t)^3$
   c. $T_{go} = 1.6$ s
4. Satisfied the boundary conditions

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Description of E Guidance Extensions: Rotational

Extend E Guidance to include angular acceleration commands [2-4]:

$$\alpha_\phi = c_1\tau^2 + c_2\tau^3 - \frac{I_{yy} - I_{zz}}{I_{xx}}\omega_y\omega_z$$

$$\alpha_\theta = c_3\tau^2 + c_4\tau^3 - \frac{I_{zz} - I_{xx}}{I_{yy}}\omega_x\omega_z$$

$$\alpha_\psi = c_5\tau^2 + c_6\tau^3 - \frac{I_{xx} - I_{yy}}{I_{zz}}\omega_x\omega_y$$

- $\alpha_\phi, \alpha_\theta, \alpha_\psi$: angular acceleration commands
- $\omega_x, \omega_y, \omega_z$: body angular velocity
- I_{xx}, I_{yy}, I_{zz}: diagonal components of symmetric inertia matrix (principal axes align with body axes)

[2] Kawamura, E.: Integrated targeting, guidance, navigation, and control for unmanned aerial vehicles. Ph.D. dissertation, University of Hawai'i at Manoa (2020).
[3] Kawamura, E., Azimov, D.: Integrated targeting, guidance, navigation, and control for unmanned aerial vehicles. In: Volume 168 of the Advances in the Astronautical Sciences Series, pp. 4259–4277. Univelt (2019)
[4] Kawamura, E., Azimov, D.: Extremal control and modified explicit guidance for autonomous unmanned aerial vehicles. Journal of Autonomous Vehicles and Systems 2(1) (2022)

Description of E Guidance Extensions: Desired Intermediate Positions & Velocities

1. Augment $\boldsymbol{E}, \boldsymbol{F} \rightarrow \boldsymbol{E}, \boldsymbol{F} \in \mathbb{R}^{4\times 4}$ with $\boldsymbol{E} = \boldsymbol{F}^{-1}$
2. Consider the intermediate positions and velocities halfway during the maneuver: $T/2$
3. First two rows of $\boldsymbol{F}$ are intermediate conditions
4. Last two rows of $\boldsymbol{F}$ are terminal conditions

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{F1}_{2\times 2} & \boldsymbol{0}_{2\times 2} \\ \boldsymbol{0}_{2\times 2} & \boldsymbol{F2}_{2\times 2} \end{bmatrix} \quad (6)$$

$$\boldsymbol{F1} = \begin{bmatrix} \int_{t_0}^{\boxed{T/2}} p_1(t)dt & \int_{t_0}^{\boxed{T/2}} p_2(t)dt \\ \int_{t_0}^{\boxed{T/2}} \left[\int_{t_0}^{t} p_1(s)ds\right] dt & \int_{t_0}^{\boxed{T/2}} \left[\int_{t_0}^{t} p_2(s)ds\right] dt \end{bmatrix}$$

$$\boldsymbol{F2} = \begin{bmatrix} \int_{t_0}^{\boxed{T}} p_1(t)dt & \int_{t_0}^{\boxed{T}} p_2(t)dt \\ \int_{t_0}^{\boxed{T}} \left[\int_{t_0}^{t} p_1(s)ds\right] dt & \int_{t_0}^{\boxed{T}} \left[\int_{t_0}^{t} p_2(s)ds\right] dt \end{bmatrix} \quad (7)$$

Description of E Guidance Extensions: Desired Intermediate Positions & Velocities

1. Vector with desired intermediate and final position and velocity conditions

$$x_{I,F} = \begin{bmatrix} \dot{x}\left(\frac{T}{2}\right) - \dot{x}(t_0) \\ x\left(\frac{T}{2}\right) - x(t_0) - \dot{x}(t_0)T_{go/2} \\ \dot{x}(T) - \dot{x}\left(\frac{T}{2}\right) \\ x(T) - x\left(\frac{T}{2}\right) - \dot{x}\left(\frac{T}{2}\right)T_{go/2} \end{bmatrix} \quad (8)$$

2. Differences from Ref. [2]
   a. Integrate two linearly independent polynomials
   b. Use the intermediate conditions for the final conditions (last two rows)
3. Preliminary tests → method does not satisfy the boundary conditions, position diverges

[2] Kawamura, E.: Integrated targeting, guidance, navigation, and control for unmanned aerial vehicles. Ph.D. dissertation, University of Hawai'i at Manoa (2020).

Description of E Guidance Extensions: Final Desired Jerk Vector

1. Four linearly independent functions: p_1, p_2, p_3, p_4
2. First & second conditions come from the column vector on RHS of Eqn. (4)
3. Third & fourth conditions: final desired acceleration & jerk, i.e., Eqn. (10)
4. Algebraically solve for the coefficients without $\boldsymbol{E}, \boldsymbol{F}$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \begin{bmatrix} \dot{x}_D - \dot{x}_0 \\ x_D - x_0 - \dot{x}_0 T_{go} \end{bmatrix} \quad (4)$$

$$\boldsymbol{a_x = c_1 p_1(t) + c_2 p_2(t) + c_3 p_3(t) + c_4 p_4(t)}$$
$$\boldsymbol{j_x = \frac{da_x}{dt} = c_1 p'_1(t) + c_2 p'_2(t) + c_3 p'_3(t) + c_4 p'_4(t)} \quad (10)$$

- Four unknown coefficients: $\boldsymbol{c_1, c_2, c_3, c_4}$
- Four equations with conditions from Eqns. (4,10)
- Resembles Cherry's method for final desired attitude

Description of E Guidance Extensions: Four Integrations of the F Matrix

1. Four integrations of four polynomials: p_1, p_2, p_3, p_4
2. Desired acceleration and jerk are the 3rd & 4th conditions
3. Boundary conditions are not satisfied: position, velocity, & jerk

$$\boldsymbol{F_{4\times4}} = \begin{bmatrix} \int p_1(t)dt & \dots & \int p_4(t)dt \\ \int\int p_1(s)dsdt & \dots & \int\int p_4(s)dsdt \\ \int\int\int p_1(u)dudsdt & \dots & \int\int\int p_4(u)dudsdt \\ \int\int\int\int p_1(r)drdsdudt & \dots & \int\int\int\int p_4(r)drdsdudt \end{bmatrix} \quad (11)$$

$$\boldsymbol{c_x} = \boldsymbol{E_{4\times4}} \begin{bmatrix} \dot{x}_D - \dot{x}_0 \\ x_D - x_0 - \dot{x}_0 T_{go} \\ \boldsymbol{a_{xD}} + \boldsymbol{g_x} \\ \boldsymbol{j_{xD}} \end{bmatrix} \quad (12)$$

Description of E Guidance Extensions: Three Integrations of the F Matrix

1. Resembles previous method but has only three integrations & removes final desired jerk
2. Promising results: usually satisfies position and velocity boundary conditions but not final desired acceleration
3. Most promising method but sometimes has an initial reverse

$$\boldsymbol{F_{3\times3}} = \begin{bmatrix} \int p_1(t)dt & \int p_2(t)dt & \int p_3(t)dt \\ \int\int p_1(s)dsdt & \int\int p_2(s)dsdt & \int\int p_3(s)dsdt \\ \int\int\int p_1(u)dudsdt & \int\int\int p_2(u)dudsdt & \int\int\int p_3(u)dudsdt \end{bmatrix} \quad (13)$$

$$\boldsymbol{c_x} = \boldsymbol{E_{3\times3}} \begin{bmatrix} \dot{x}_D - \dot{x}_0 \\ x_D - x_0 - \dot{x}_0 T_{go} \\ \boldsymbol{a_{xD}} + \boldsymbol{g_x} \end{bmatrix} \quad (14)$$

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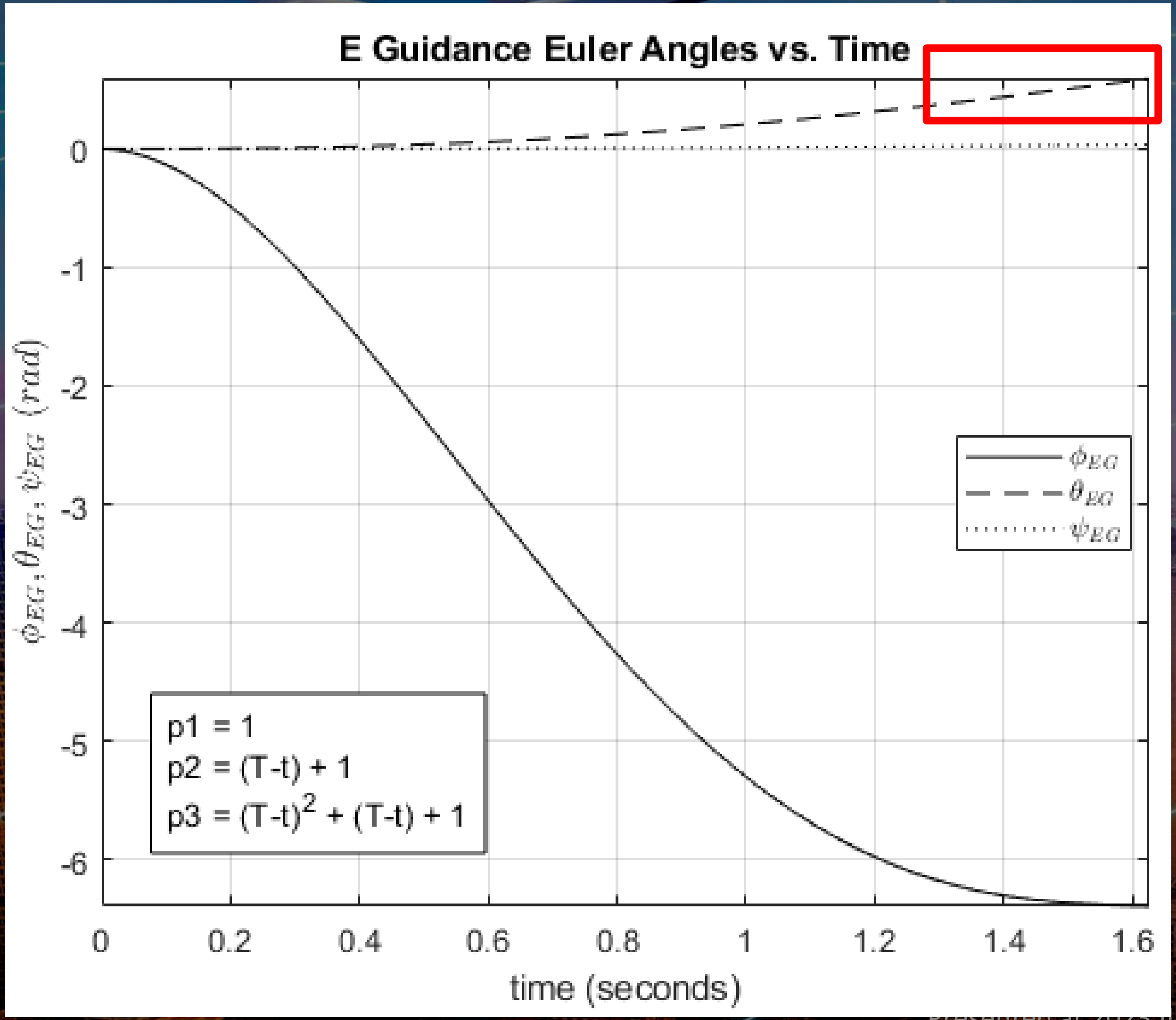
Simulation Results

1. Three quadcopter unmanned aerial vehicle maneuvers
   a. 360° Roll
   b. Vertical takeoff (1D)
   c. Waypoint (3D)
2. Utilize the E Guidance method with three integrations (most promising method)

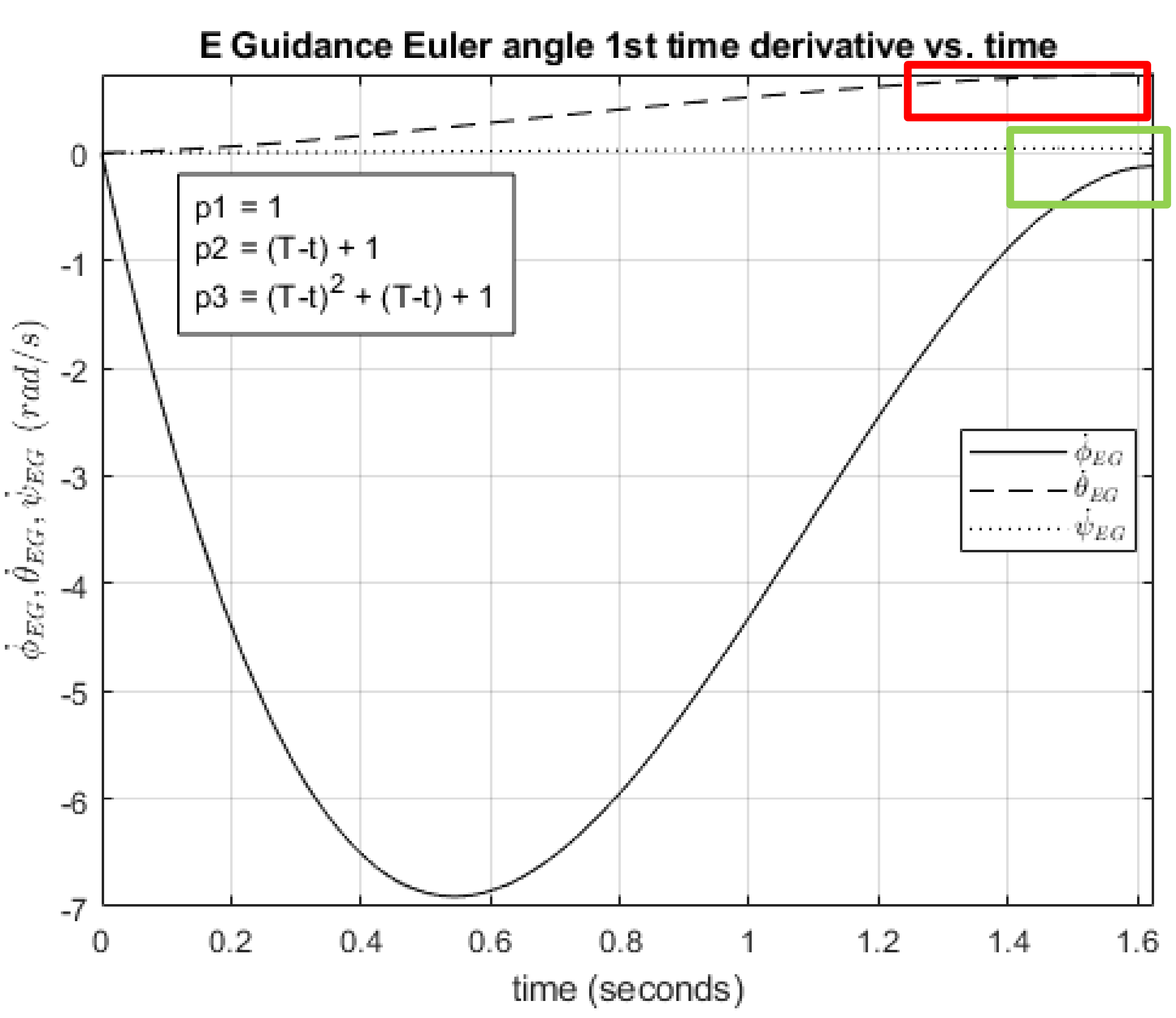
Simulation Results: 360° Roll Maneuver



1. Boundary conditions
   a. Initial attitude: [0,0,0]
   b. Desired attitude: [-2π,$\boxed{0}$,0]
2. Use final desired 2nd time derivative of Euler angles as 3rd condition
3. No initial backwards motion
4. Slight divergence in θ (pitch): 0.595 rad or 34.1° (desired 0°)

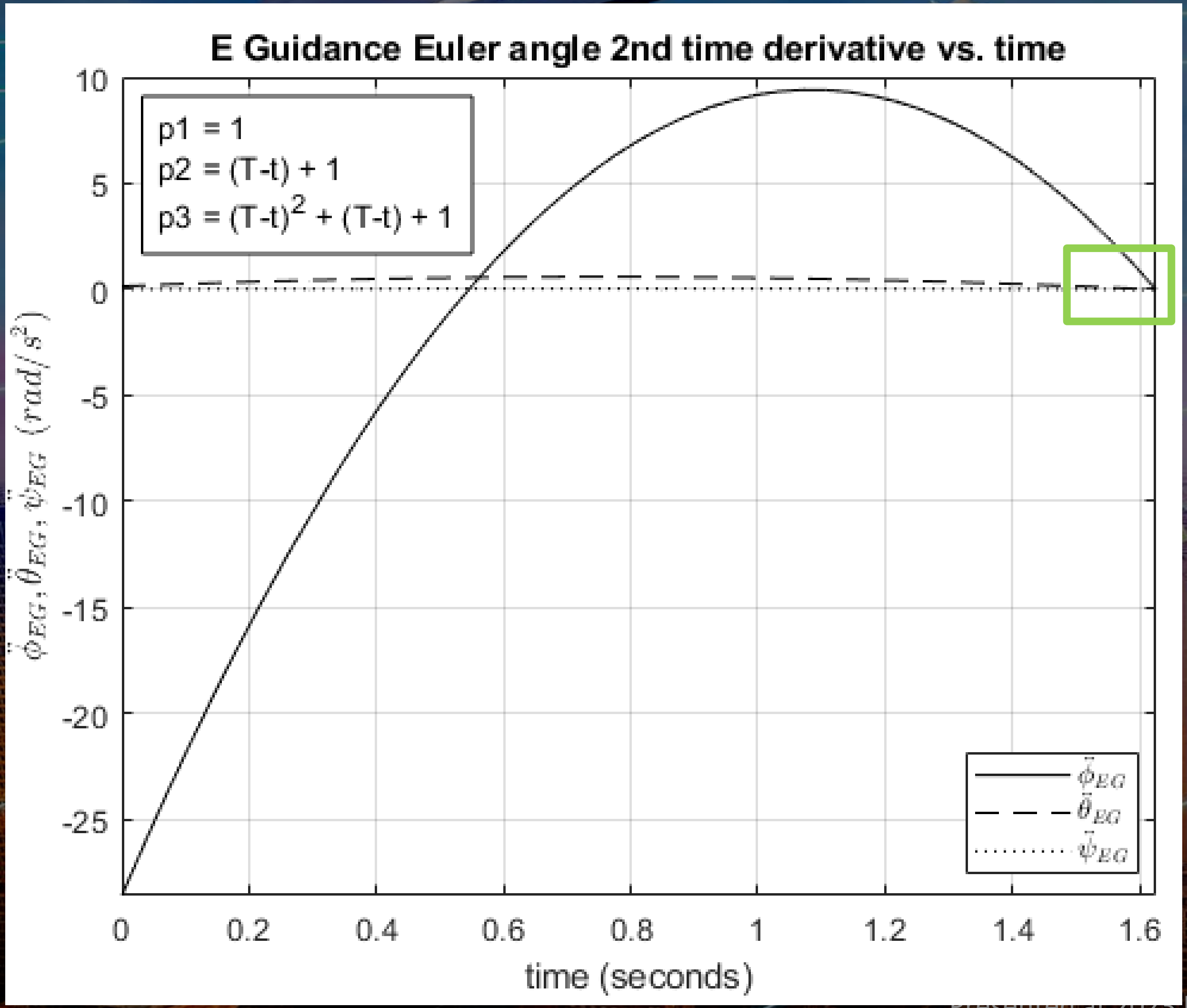
$$\boldsymbol{c_\phi} = \boldsymbol{E_{3\times3}} \begin{bmatrix} \dot{\phi}_D - \dot{\phi}_0 \\ \phi_D - \phi_0 - \dot{\phi}_0 T_{go} \\ \boxed{\ddot{\boldsymbol{\phi}}_{\boldsymbol{D}}} \end{bmatrix} \quad (15)$$

Simulation Results: 360° Roll Maneuver



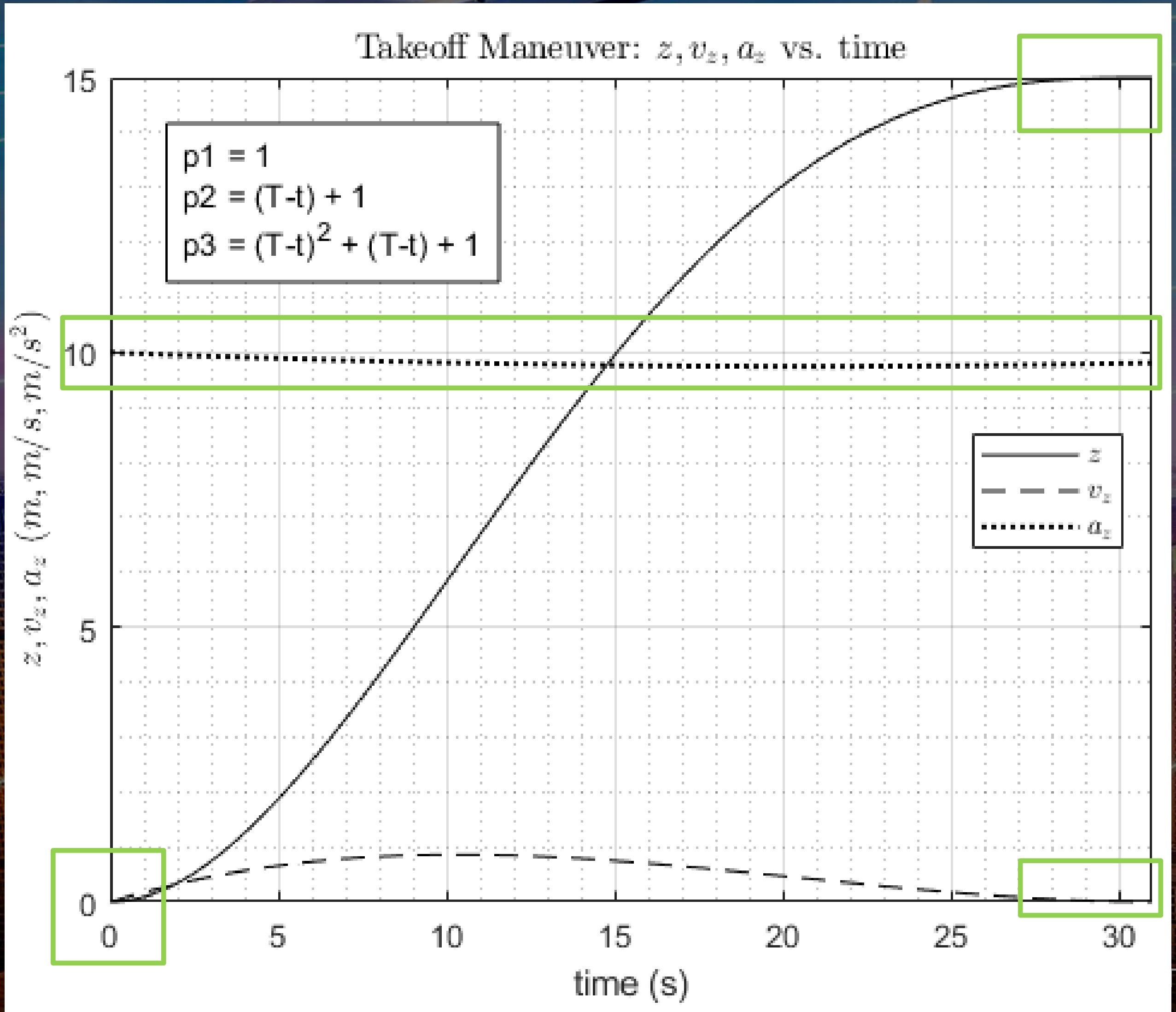
1. Boundary conditions
   a. Initial angular velocity: [0,0,0]
   b. Desired angular velocity: [0,0,0]
2. Slight divergence in $\dot{\theta}$ (pitch velocity)
3. Roll and yaw velocity converge to zero

Simulation Results: 360° Roll Maneuver



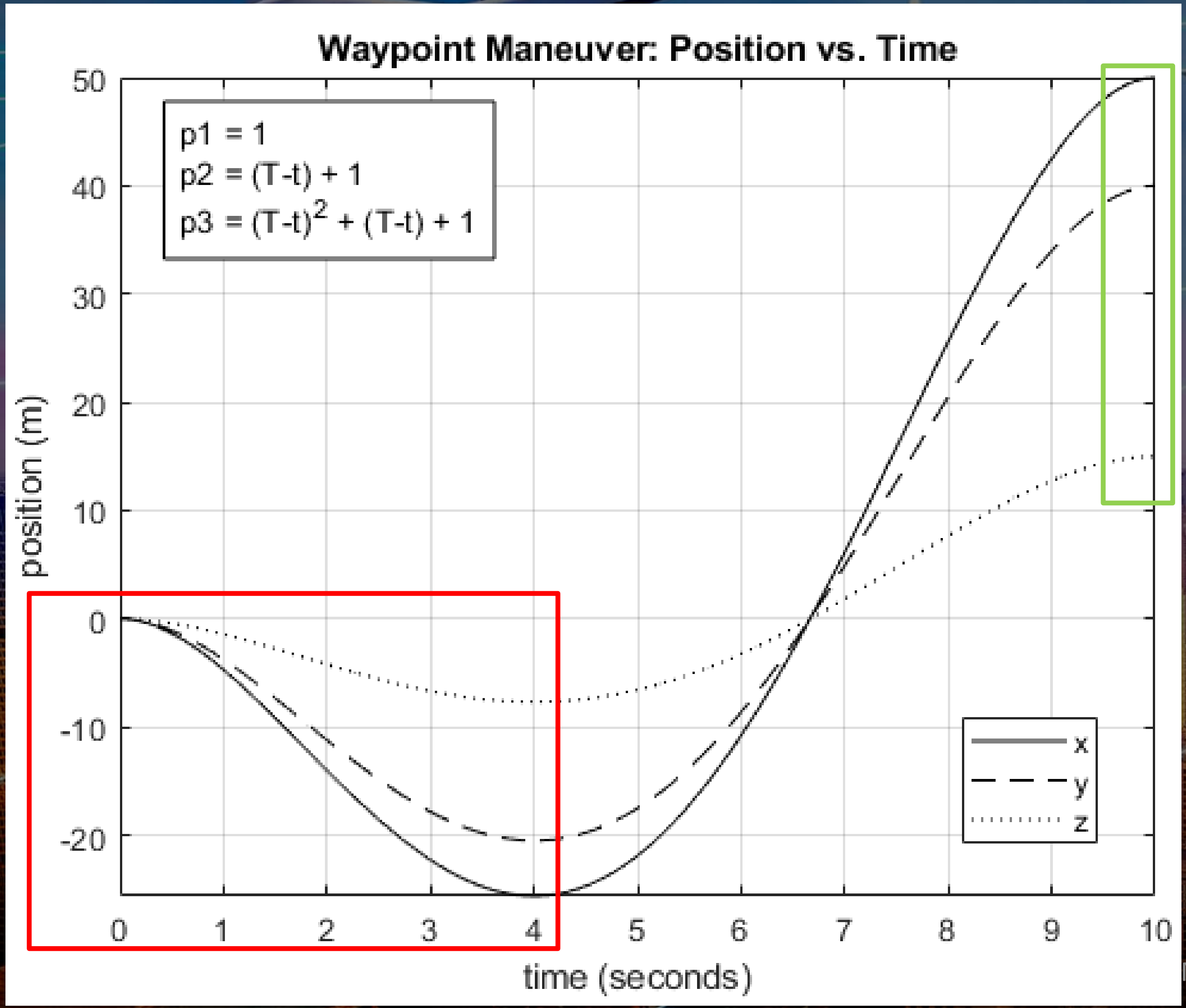
1. Final desired Euler angular acceleration: [0,0,0]
2. Final accelerations converge to zero

Simulation Results: Takeoff Maneuver



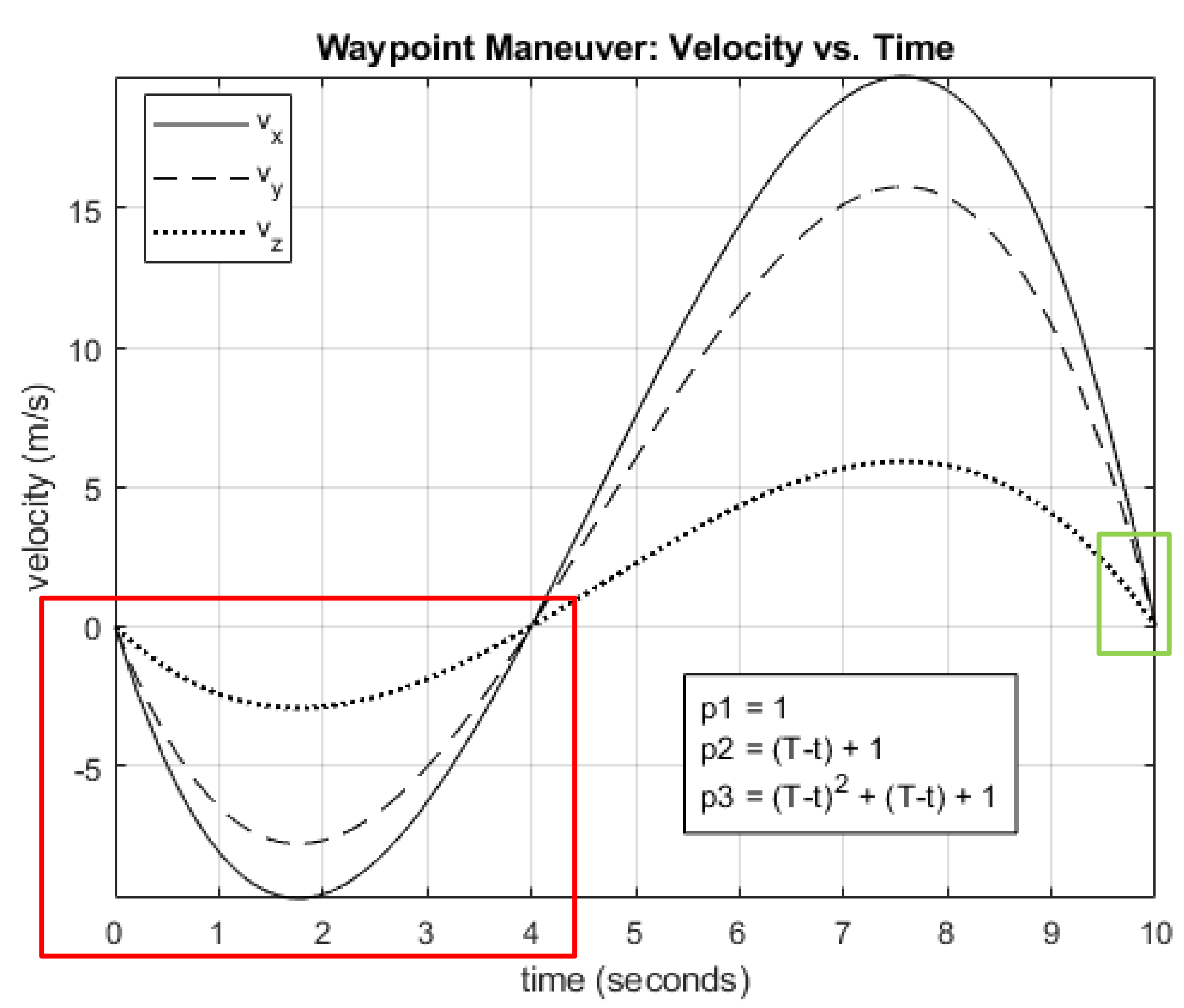
1. Boundary conditions
   a. Initial altitude: 0 m
   b. Final desired altitude: 15 m
   c. Zero initial and final velocity
2. No initial reverse
3. All boundary conditions satisfied
4. Acceleration ~10 m/s^2 due to gravity

Simulation Results: Waypoint Maneuver



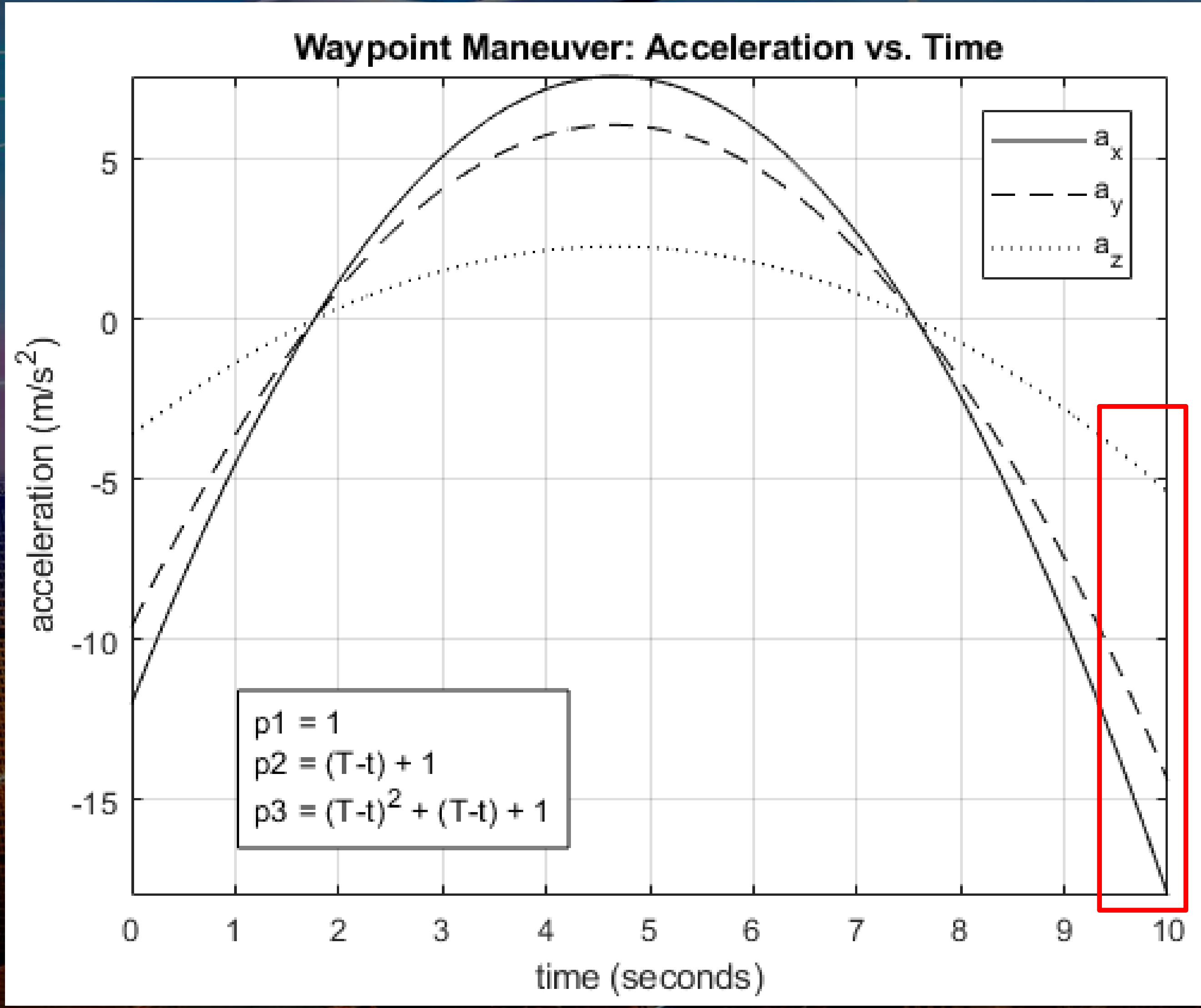
1. Position boundary conditions
   a. Initial: [0,0,0]
   b. Final desired: [50,40,15]
2. Initial reverse
3. Final desired position satisfied

Simulation Results



1. Velocity boundary conditions
   a. Initial: [0,0,0]
   b. Final desired: [0,0,0]
2. Initial reverse velocity
3. Final desired velocity satisfied

Simulation Results



1. Non-zero final desired acceleration
2. Numerous combinations did not satisfy the boundary conditions, diverged, or oscillated
   a. Exponential
   b. Sinusoidal
   c. Higher-order polynomials
   d. See Appendix A of Ref. [2]

[2] Kawamura, E.: Integrated targeting, guidance, navigation, and control for unmanned aerial vehicles. Ph.D. dissertation, University of Hawai'i at Manoa (2020).

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Conclusion

1. Four proposed extensions for E Guidance
   a. Intermediate position and velocity ✗
   b. Jerk vector and neglected $\boldsymbol{E}$ matrix ✗
   c. Four integrations of the $\boldsymbol{F}$ matrix ✗
   d. Three integrations of the $\boldsymbol{F}$ matrix (most promising) ✓
      i. Initial reverse may be impractical for
         a) Fixed-wing aircraft
         b) Obstacles behind or below
      ii. Beneficial for reversing from incoming target with waypoint/target in front
2. Overall the 2×2 formulation works best
   a. Counterintuitive if thinking about TSE → increased accuracy with higher dimensions
   b. More research is needed to select $p_i(t)$ & satisfy the boundary conditions
3. Three of four methods do not satisfy the boundary conditions but presented to prevent future researchers from reinventing inadequate methods

References

1. Cherry, G.: A general, explicit, optimizing guidance law for rocket-propelled spaceflight. In: Astrodynamics Guidance and Control Conference, p. 638 (1964)
2. Kawamura, E.: Integrated targeting, guidance, navigation, and control for unmanned aerial vehicles. Ph.D. dissertation, University of Hawai'i at Manoa (2020)
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4. Kawamura, E., Azimov, D.: Extremal control and modified explicit guidance for autonomous unmanned aerial vehicles. Journal of Autonomous Vehicles and Systems 2(1) (2022)

Acknowledgements

1. IUTAM Scientific Committee: GA18-13: Optimal Guidance and Control for Autonomous Systems
2. IUTAM Organizing Committee
3. Dilmurat Azimov, IUTAM Chair of the Scientific Committee

Mahalo! Any questions?